Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - IV

Subject Code : 4TE04EMT1 **Branch: B.Tech (Auto,Civil,EE,EC,Mech)**

Semester : 4 **Date : 20/10/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) δ equal to

(A) $\dfrac{\Delta}{E^{\frac{1}{2}}}$ (B) $E^{\frac{1}{2}} + E^{\frac{-1}{2}}$ (C) $E^{\frac{1}{2}} - E^{\frac{-1}{2}}$ (D) none of these

b) $\Delta\nabla$ equal to

(A) $\nabla + \Delta$ (B) $\nabla - \Delta$ (C) $\nabla\Delta$ (D) none of these

c) Putting $n=1$ in the Newton – Cote's quadrature formula following rule is obtained

(A) Simpson's rule (B) Trapezoidal rule (C) Simpson's $\dfrac{3}{8}$ rule

(D) none of these

d) In application of Simpson's $\dfrac{1}{3}$ rule, the interval of integration for closer approximation should be

(A) odd and small (B) even and small (C) even and large (D) none of these

e) The Gauss elimination method in which the set of equations are transformed into triangular form.

(A) TRUE (B) FALSE

f) Jacobi iteration method can be used to solve a system of non – linear equations.

(A) TRUE (B) FALSE

g) The auxiliary quantity k_1 obtained by Runge – Kutta fourth order for the differential equation $\dfrac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, when $h = 0.1$ is

(A) 0.1 (B) 0 (C) 1 (D) none of these

h) The first approximation y_1 of the initial value problem $\dfrac{dy}{dx} = x^2 + y^2$, $y(0) = 0$ obtain by Picard's method is

(A) x^2 (B) $\frac{x^2}{2}$ (C) $\frac{x^3}{3}$ (D) none of these

i) The Fourier cosine transform of $f(x)=5e^{-2x}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2+4}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{2}{\lambda^2+4}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2-4}\right)$ (D) none of these

j) The Fourier sine transform of $f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{1+\cos a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda^2}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda}\right)$

(D) none of these

k) The function $2x-x^2+py^2$ is harmonic if p equal to

(A) 0 (B) 1 (C) 2 (D) 3

l) Under the transformation $w=\frac{1}{z}$ the image of $|z-2i|=2$ is

(A) $v=\frac{1}{4}$ (B) $v=\frac{-1}{4}$ (C) $|w-2i|=2$ (D) $u^2+v^2=4$

m) The tangent vector at the point $t=1$ on the curve $x=t^2+1, y=4t-3, z=t^3$ is

(A) $2i-4j+3k$ (B) $2i+4j+3k$ (C) $2i-4j-3k$ (D) $2i+4j-3k$

n) If $\vec{V}=(3xyz)i-(2x^2y)j+(2z)k$ then $|\text{div}\,\vec{V}|$ at $(1,1,1)$ is

(A) 0 (B) 3 (C) 1 (D) 2

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Given $\sin 45^\circ=0.7071, \sin 50^\circ=0.7660, \sin 55^\circ=0.8192, \sin 60^\circ=0.8660$, find $\sin 52^\circ$, using Newton's forward interpolation formula. **(5)**

b) Given **(5)**

x:	10	20	30	40	50
y:	600	512	439	346	243

Using Stirling's formula find y_{35}.

c) Find the finite Fourier sine transform of $f(x)=lx-x^2, \quad 0\le x\le l$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations by Gauss-Seidal method. **(5)**

$27x+6y-z=85,\ 6x+5y+2z=72,\ x+y+54z=110$

b) Given that **(5)**

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Find $\frac{dy}{dx}$ at $x=1.05$.

c) Determine the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Use the fourth – order Runge Kutta method to solve $\frac{dy}{dx} = y - \frac{2x}{y}; \quad y(0) = 1$. **(5)**

Evaluate the value of y when x = 0.2 and 0.4

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 3/8 Rule using $h = \frac{1}{6}$. **(5)**

c) Solve the following system of equations by Gauss-Jordan Method: **(4)**

$5x - 2y + 3z = 18,\ x + 7y - 3z = -22,\ 2x - y + 6z = 22$

Q-5 **Attempt all questions** **(14)**

a) If $f(z) = f(re^{i\theta}) = P(r,\theta) + iQ(r,\theta)$ is an analytic function, prove that both P and Q satisfy the Laplace equation in polar coordinates, namely **(5)**

$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0$.

b) If $\phi = 45x^2y$, then evaluate $\iiint_V \phi\, dV$, where V denote the closed region bounded by the planes $4x + 2y + z = 8,\ x = 0,\ y = 0,\ z = 0$. **(5)**

c) Use Lagrange's Interpolation Formula to find the value of y when x = 3.5, if the following values of x and y are given: **(4)**

x	1	2	3	4
y	1	8	27	64

Q-6 **Attempt all questions** **(14)**

a) If $\vec{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2xz)\hat{k}$, show that $\vec{F}$ is irrotational but not solenoidal. **(5)**

b) Under the transformation $w = \frac{1}{z}$ **(5)**

(a) Find the image of $|z - 2i| = 2$

(b) Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscates $\rho^2 = \cos 2\theta$.

c) Using Taylor's series method, compute $y(-0.1)$, $y(0.1)$, $y(0.2)$ correct to four decimal places, given that $\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Show that the function defined by the equation **(5)**

$$f(z) = \begin{cases} u(x,y) + iv(x,y), & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$$

where $u(x,y) = \frac{x^3 - y^3}{x^2 + y^2}$ and $v(x,y) = \frac{x^3 + y^3}{x^2 + y^2}$ is not analytic at $z = 0$ although Cauchy – Riemann equations are satisfied at that poiut.

b) Using Green's Theorem, evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. **(5)**

c) Evaluate $\int_0^1 x^3\, dx$ by Trapezoidal Rule using 5 subintervals. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Given $\frac{dy}{dx} = xy$ with $y(1) = 5$. Using Euler's method find the solution correct to three decimal position in the interval $[1, 1.5]$ taking step size $h = 0.1$. **(5)**

b) Using Fourier integral show that $\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$ **(5)**

c) Prove that the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$ is $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$. **(4)**